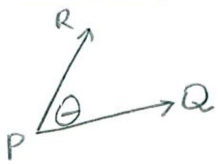
Let P be the point $(-3, 4, -1)$, R be the point $(-5, 3, 1)$, and $\overrightarrow{PQ}$ be the vector $4\vec{i} - \vec{j} - \vec{k}$. **SCORE: ____ / 103 PTS**

3 EACH EXCEPT AS NOTED

[a] If $\overrightarrow{PR}$ is parallel to $< -3, 2-c, b+1 >$, find the value of b.

$$\overrightarrow{PR} = \langle -2, -1, 2 \rangle = k\langle -3, 2-c, b+1 \rangle \quad 4$$
$$= \langle -3k, (2-c)k, (b+1)k \rangle$$
$$-2 = -3k \qquad 2 = (b+1)k$$
$$k = \tfrac{2}{3} \qquad 2 = \tfrac{2}{3}(b+1) \longrightarrow b = 2$$

[b] Find $\angle RPQ$.



$$\cos^{-1}\frac{\overrightarrow{PR}\cdot\overrightarrow{PQ}}{\|\overrightarrow{PR}\|\|\overrightarrow{PQ}\|} = \cos^{-1}\frac{-8+1-2}{(3)(3\sqrt{2})} = \cos^{-1}\frac{-9}{9\sqrt{2}}$$
$$= \cos^{-1}\frac{-\sqrt{2}}{2}$$
$$= \frac{3\pi}{4} \text{ OR } 135^\circ$$

[c] Find the area of triangle PQR.

$$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & -1 \\ -2 & -1 & 2 \end{vmatrix}\begin{matrix} \vec{i} & \vec{j} \\ 4 & -1 \\ -2 & -1 \end{matrix} = \langle -3, -6, -6 \rangle \quad 6$$

CHECK: $\langle -3,-6,-6\rangle\cdot\langle 4,-1,-1\rangle$
$= -12+6+6 = 0$
$\langle -3,-6,-6\rangle\cdot\langle -2,-1,2\rangle$
$= 6+6-12 = 0$ ✓

$$\tfrac{1}{2}\|\langle -3,-6,-6\rangle\| = \tfrac{1}{2}|-3|\|\langle 1,2,2\rangle\|$$
$$= \tfrac{1}{2}\cdot 3\cdot 3$$
$$= \tfrac{9}{2}$$

[d] Find the coordinates of Q.

$$\langle x+3, y-4, z+1 \rangle = \langle 4, -1, -1 \rangle$$

$x+3 = 4$
$y-4 = -1$
$z+1 = -1$

$$(x, y, z) = (1, 3, -2)$$

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[e] Find the general **(NOT point-normal)** equation of the plane which contains P, Q and R.

$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -3, -6, -6 \rangle$ or $\langle 1, 2, 2 \rangle$ 6

$(x+3) + 2(y-4) + 2(z+1) = 0$

$x + 2y + 2z - 3 = 0$

+3 SIMPLIFY

[f] Find parametric equations of the line which is perpendicular to the plane in part [e], and also contains Q.

$\vec{d} = \vec{n}$

$x = 1 + t$
$y = 3 + 2t$
$z = -2 + 2t$ 6

[g] Find symmetric equations of the line which is parallel to the line in part [f], and also is perpendicular to the plane in part [e], and also contains P.

$\vec{d_2} = \vec{d} = \vec{n}$

$\dfrac{x+3}{1} = \dfrac{y-4}{2} = \dfrac{z+1}{2}$ 6

$x + 3 = \dfrac{y-4}{2} = \dfrac{z+1}{2}$

[h] Find a vector of magnitude 8 perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$\dfrac{8}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}(\overrightarrow{PQ} \times \overrightarrow{PR}) = \dfrac{8}{9}\langle -3, -6, -6 \rangle$

$= \langle -\frac{8}{3}, -\frac{16}{3}, -\frac{16}{3} \rangle$ or $\langle \frac{8}{3}, \frac{16}{3}, \frac{16}{3} \rangle$

Find all octants in which $yz < 0$ and $x < 0$ simultaneously. **SCORE: ____ / 10 PTS**

2 $x < 0,\ y < 0,\ z > 0 \longrightarrow O_3$ $2\frac{1}{2}$

2 $x < 0,\ y > 0,\ z < 0 \longrightarrow O_{2+4}$ or O_6 $3\frac{1}{2}$

Fill in the blanks. List all correct answers. **SCORE: ____ / 12 PTS**

[a] If $\vec{u} \cdot \vec{u} = 10$, then $\|\vec{u}\| =$ $\underline{\sqrt{10}}$ and $\vec{u} \times \vec{u} =$ $\underline{\vec{0}}$.

2 EACH

$\langle 0, 3, -2 \rangle = \langle 4-x, -1-y, -8-z \rangle$

[b] If the terminal point of $\vec{v} = 3\vec{j} - 2\vec{k}$ is $(4, -1, -8)$, then the initial point of $\vec{v}$ is $\underline{(4, -4, -6)}$.

[c] The equation of the $yz-$plane is $\underline{x=0}$ and the equation of the $z-$axis is $\underline{x=y=0}$.

[d] If you start at the point $(-1, -5, 1)$, then move 3 units upward, 8 units to the left and 6 units backward,

you will be at the point $\underline{(-7, -13, 4)}$. $(-1-6, -5-8, 1+3)$

Consider the sphere $x^2 + y^2 + z^2 - 8x + 12y + 14z + 65 = 0$. **SCORE: ____ / 15 PTS**

[a] Find the equation of the $xz-$trace. Describe the $xz-$trace.

$$x^2 - 8x + 16 + y^2 + 12y + 36 + z^2 + 14z + 49 = \underline{-65 + 16 + 36 + 49}$$

$$\underline{(x-4)^2 + (y+6)^2 + (z+7)^2 = 36}$$

$1\frac{1}{2}$ $\underline{y=0} \rightarrow (x-4)^2 + 36 + (z+7)^2 = 36$

$$\underline{(x-4)^2 + (z+7)^2 = 0}$$

$\underline{\text{POINT } (4, 0, -7)}$

2 EACH EXCEPT AS NOTED

[b] Find the equation of the $xy-$trace. Describe the $xy-$trace.

$1\frac{1}{2}$ $\underline{z=0} \rightarrow (x-4)^2 + (y+6)^2 + 49 = 36$

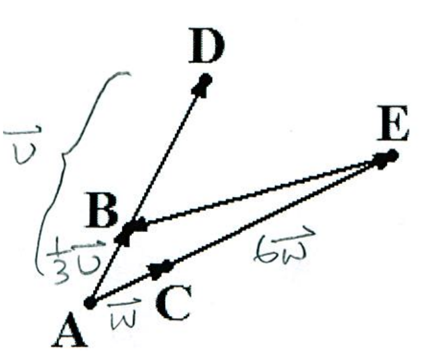
$$\underline{(x-4)^2 + (y+6)^2 = -13}$$

$\underline{\text{NO TRACE}}$

In the diagram below, ABD and ACE are both line segments. **SCORE: ____ / 10 PTS**

CE is six times the length of AC, and AD is three times the length of AB. **(NOTE: The diagram is <u>NOT</u> drawn to scale.)**

If $\vec{u} = \overrightarrow{AD}$ and $\vec{w} = \overrightarrow{AC}$, find an expression for $\overrightarrow{EB}$ in terms of $\vec{u}$ and $\vec{w}$.



$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$

$= \underbrace{-7\vec{w}}_{5} + \underbrace{\tfrac{1}{3}\vec{u}}_{5}$ OR $\frac{1}{3}\vec{u} - 7\vec{w}$